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Analytical formulas for current dynamics and peak temperature during quench in MRI

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Abstract

Quench of a superconducting magnet in MRI scanners may cause damage to the coil, but is unavoidable in some situations. Knowledge of the temporal evolution of current and the maximum temperature developed in the coil make it possible to determine the state the coil after the quench. Commonly, these parameters are computed using three-dimensional simulation programs. However, numerical models do not provide physical links between basic parameters of the magnet and quench dynamics. This paper presents analytical formulas that may serve as reasonable estimates, showing relationship between quench dynamics and parameters of a superconducting magnet.

Keywords: Superconducting coils, quench, magnet, MRI

Introduction

In recent years, great attention has been paid to the problem of simulation and assessment of aftermaths of quench in MRI. When quench occurs in a superconducting magnet, it is not always clear if the coil requires repairs or not. If it was damaged, then the magnet cannot be ramped up, and lengthy and expensive repairs are in order. However, if the probability of coil damage is minimal, then re-initialization is the optimal, cost-effective choice. Due to that, numerous cumbersome 3D simulation programmes have been created that enable the computation of quench dynamics in time and space, and allow estimation of values of current and peak temperature. Such programmes depend on many numerical parameters, which need to be known before the program can be executed. Sometimes, even little errors in the values of these parameters may lead to significant deviations. Therefore, a simple qualitative theory with relatively easily computable formulas, that can be used to estimate quench dynamics, would present an advantage in determining the state of the coil. In the present short communication such formulas are derived with very reasonable simplifications.

Model

In the simplified model, only lateral propagation of quench will be considered. Lateral propagation of quench is the spread of temperature increase along the superconducting wire. Thus, the effect that the rise of local temperature has on the surrounding wires is neglected. The reason for this assumption is the high velocity of quench propagation, about 2 m/s for 240 A coils, and the design of the superconducting coil, in which each

superconducting wire is separated from the neighboring wires by layers of supporting and shielding materials.

Figure 1 explains the model. Non-linear thermal dependence of the specific resistance r around critical temperature T_c is the principle phenomenon of the quench. What is essential, that it is practically zero below T_c, and abruptly rises to the non-zero value r above this temperature. The exact temperature dependence of r when $T > T_c$ is not important because it gives only second-order corrections. We shall assume, therefore,

$$r = \begin{cases} 0, & T < T_c \\ r, & T > T_c \end{cases}.$$

(1)

This approximation may be called the bar-step approximation. Before the quench starts, the wire transmits initial current I_0. We assume that the quench originates at $x = x_0$ at the moment $t = 0$. At this moment, specific resistance rises from zero to $r > 0$, and electrical power

$$\frac{r\,dx}{S}I^2$$

(2)

begins to develop within this element of wire. Thus, variation of the heat during time increment dt is

$$dQ = \frac{r\,dx}{S}I^2 \cdot dt.$$

(3)

As the temperature at $x = x_0$ rises due to this heat, heat flux spreads out of the volume $dV = S.dx$ due to temperature
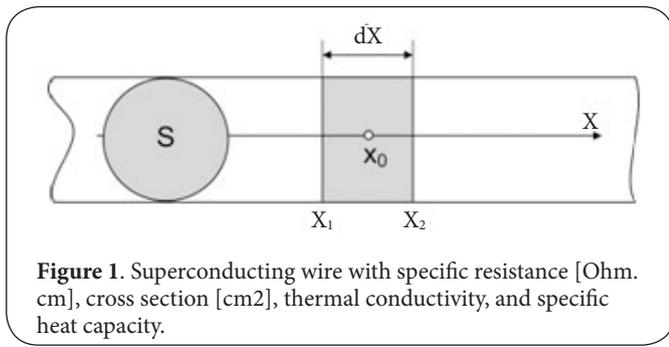
Figure 1. Superconducting wire with specific resistance [Ohm. cm], cross section [cm2], thermal conductivity, and specific heat capacity.

gradient dT/dx. This leads to basic equation:

$$\frac{r}{S^2}I^2 + \frac{\partial}{\partial x}\left(k(x)\frac{\partial T}{\partial x}\right) = \rho\, C\frac{dT}{dt}. \qquad (4)$$

where $k(x)$ is thermal conductivity and ρ - density of the superconducting wire. Note that the second term in the left-hand side acts negatively because the second derivative of temperature around x_0 is negative.

Solution for the current

Equation (4) has to be completed by the electrical equation, which follows from the energy conservation law: the sum of the magnetic energy E stored in the magnet and the power transferred into heat Q is constant:

$$E + Q = const\ ;\ -dE = \delta Q \qquad (5)$$

Here δQ is the total dissipation along the entire length of the superconducting wire:

$$\delta Q = dt \cdot I^2 \frac{1}{S}\int r(x)\,dx. \qquad (6)$$

According to phenomenology of quench, we introduce the longitudinal velocity of quench propagation v_0 [cm/s]. With it, the length l of the wire section corrupted by quench is

$$l = v_0 t. \qquad (7)$$

Since other sections remain superconducting with $r = 0$, the integral in the equation above is equal to $r \cdot v_0 t$:

$$\delta Q = dt \cdot I^2 \frac{r \cdot v_0}{S}t. \qquad (8)$$

Magnetic energy is

$$E = \frac{1}{2}LI^2, \qquad (9)$$

where L is the magnet inductance, and

$$dE = L \cdot I\,dI, \qquad (10)$$

which makes the equation

$$-LI\,dI = I^2\frac{r \cdot v_0}{S}t\,dt. \qquad (11)$$

It gives simple analytical formula for evolution of the current during quench:

$$I = I_0 e^{-(t/\tau)^2},\ \ \tau = \sqrt{\frac{2SL}{rv_0}}. \qquad (12)$$

We introduced parameter τ that may be called the quench time-the time interval during which the current drops e times from the value of stable operation. The resistance per unit length

$$\eta = \frac{r}{S}\ \text{[Ohm/m]} \qquad (13)$$

is always known: it is one of the design parameters of the magnet wire that is always carefully characterized. Typically, $\eta \approx 0.15$ Ohm/m [1-4]. With this, the quench time transforms to

$$\tau = \sqrt{\frac{2L}{\eta v_0}}. \qquad (14)$$

Dimension of the right-hand side is time since inductance L measured in Henry is equal to

$$H = \frac{m^2\,kg}{C^2} = \frac{m^2\,kg}{s^2\,A^2} = \frac{J}{A^2} = \frac{Wb}{A} = \frac{V \cdot s}{A} = \frac{s^2}{F} = \Omega \cdot s,$$

where C stands for the Coulomb, Wb - for the Weber (magnetic flux), F - for the Farad, W - for the Ohm. Formula (14) for the quench time clearly shows that the smaller the magnet inductance the shorter the quench. Alternatively, the smaller the resistance above superconducting threshold the longer the quench.

Now, we shall compare the formula for the current (12) with experimental results. Qualitatively, it is consistent with all the experimental results on quench published so far. For instance, **Figure 2** shows the Gaussian curve (12) fitted into the results published in [5], with the fitting parameters I_0 =200 A and τ =7 s.
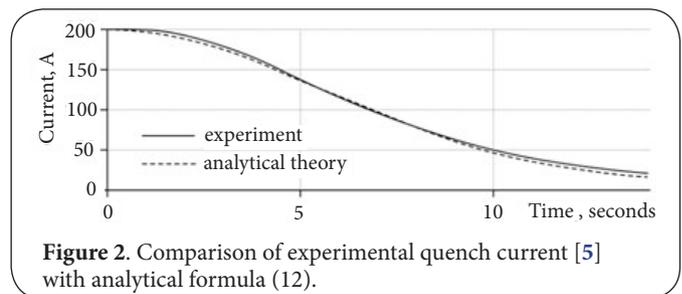

Figure 2. Comparison of experimental quench current [5] with analytical formula (12).

Whatever good the qualitative agreement is, does our theory

describe the phenomenon quantitatively? The answer to this question can only be given with the complete set of data, characterizing the superconducting magnet. One of the few publications that provide necessary data is [**6**]. Technical specifications of the magnet described in this article are reproduced in the **Table 1** below.

Table 1. Parameters of the superconducting magnet [6].

Parameters			
Dipole magnetic field	1.6 T	Average turn length	6.576 m
Usable aperture	0.38 m×0.14 m	Nominal current	232 A
N of layers×n of turns	20×28 = 560	Stored energy	414 kJ
Superconducting material	NbTi	NbTi cross section	0.173 mm^2
Cladding	Cu	Copper cross section	1.862 mm^2

It uses NbTi superconducting filaments evenly spread within the Nb wire, which is embedded into a bulk cupper strip.

Figure 3 compares experimental data for quench current from [**6**] with our theory. Analytical formula (12) is fitted into the experimental data with the value τ = 12 s. The immediate conclusion that follows from this figure is that, qualitatively, the analytical formula is again in agreement with the experiment. Next, consider quantitative details, specifically the quench velocity v_0. From (12) it follows that
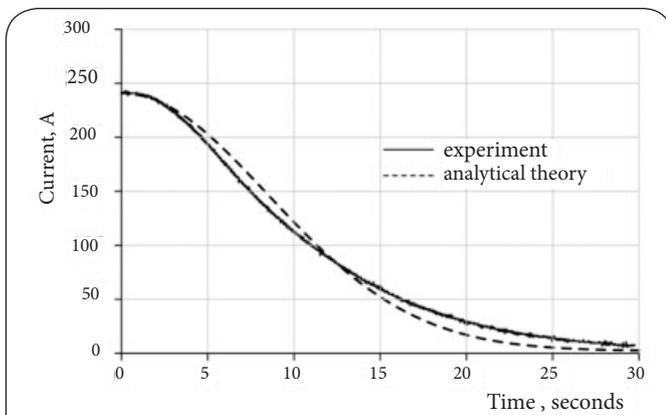

Figure 3. Comparison of experimental quench current [**6**] with analytical formula (12).

$$v_0 = \frac{2L}{\eta \tau^2} \cdot \qquad (15)$$

The table gives the value of magnetic energy stored in the magnet

$$E = \frac{1}{2}LI^2 = 414 \text{ kJ}$$

and the current I = 232 A. This is the single-coil magnet, so that we find L =15.4 H. For η we do not have necessary data, but we may take the typical value cited above: η=0.15 Ohm/m.

With all these data, formula (15) gives v_0 =1.4 m/s. Quench velocity can hardly be measured, but luckily, we can compare this value to the value numerically simulated in [**6**] and presented in **Figure 4** in the form of lines. Our value, computed for the particular current 232 A, is shown by the open circle. Thus, analytical formula (15) gives the result consistent with
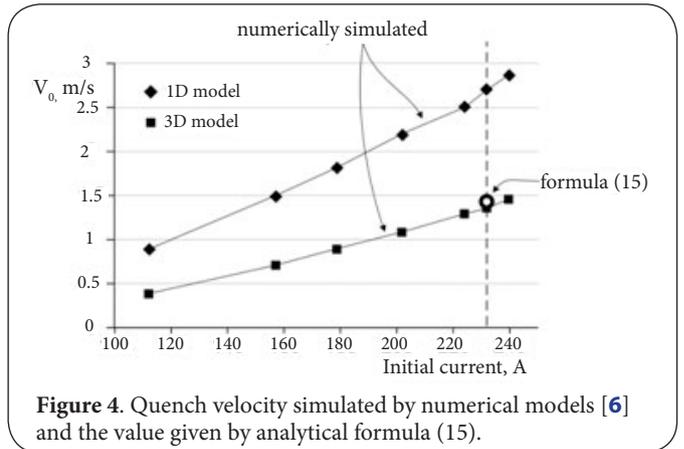

Figure 4. Quench velocity simulated by numerical models [**6**] and the value given by analytical formula (15).

the numerical models.

Solution for the temperature

Quench current and propagation velocity are, of course, important parameters, but even more important is evaluation of the maximum spot temperature that is developed during quench. If this temperature exceeds critical value, then the superconducting coil is probably damaged. Consider the basic equation (4) again. In it, specific heat capacity C is a complicated function of temperature. Copper is the main heat absorbing material of the wire, and for it $C(T)$ is shown in **Figure 5** [**4**]. With temperature being in the interval of 4 K< T <100 K, specific heat capacity of copper can be reasonably described by a second-order approximation $C = 0.03 \cdot T^2$, or in general:
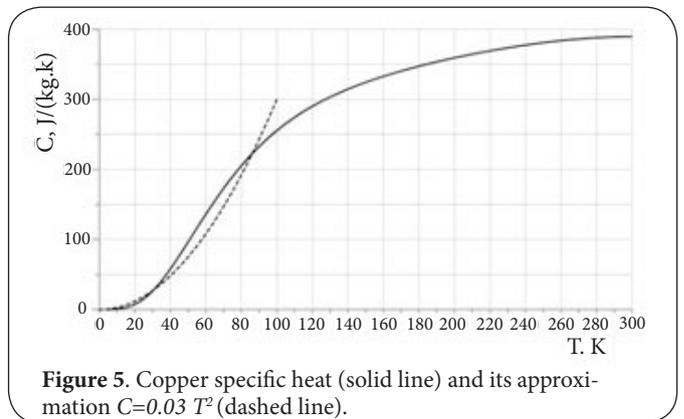
$$C = \alpha T^{\beta-1} \cdot \qquad (16)$$

Figure 5. Copper specific heat (solid line) and its approximation $C=0.03\,T^2$ (dashed line).

In such an approximation, dimension of α is J/(kg×K^b) and β is dimensionless.

The basic equation (4) is the partial differential equation of the second order that cannot be solved analytically without simplifying assumptions. First, assume thermal conductivity k independent of the temperature, and as such, independent of x. This assumption is supported by experimental phenomenology, showing that around the critical temperature T_c, k varies insignificantly [**8**].

Our main goal is to derive the formula for the maximum spot temperature. This point is located at the quench origin $x_0 = 0$, from which the quench wave spreads to the more distant sections of the wire. Here, the term with thermal conductivity k, describing heat transfer from more heated areas of the wire to the less heated ones, is insignificant and may be dropped. Indeed, after the wave of high temperature has started moving, the temperature of the adjacent regions immediately to the right and to the left from x_0 becomes almost the same as the temperature at x_0, which means that the temperature gradient in x_0 is close to zero. As such, the second derivative at $x = x_0$ may be neglected.

This leads us to the equation

$$\eta I^2\, dt = \mu\, C\, dT ,\qquad (17)$$

where again $\eta = r / S$ is the resistance of the wire per unit length, and $\mu = \rho \cdot S$ is the mass of the wire per unit length. Substituting (12) and (16) and integrating over temperature, we obtain:

$$\frac{\eta \tau I_0^2}{2}\sqrt{\frac{\pi}{2}}\, erf\!\left(\frac{t}{\tau}\sqrt{2}\right) = \mu\, p\left(T^\beta - T_0^\beta\right),\quad (18)$$

where $T_0 = 4.2$ K is the initial temperature for the super-conducting wire. This gives the solution:

$$T = \left[\frac{\eta \tau I_0^2}{2\mu p}\sqrt{\frac{\pi}{2}}\, erf\!\left(\frac{t}{\tau}\sqrt{2}\right) + T_0^\beta\right]^{1/\beta}\qquad (19)$$

Here $\quad erf(x) = \dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^x e^{-y^2}\, dy$

is the well-known error-function, and

$$p = \frac{\alpha}{\beta}\qquad (20)$$

is an important parameter of a superconducting wire, determining dynamics of the temperature. Dimension of this parameter is the same as α, i.e., J/(kg×K^b).

With t→∞, temperature stabilizes at the maximum value T_{max}. Since $erf(\infty) = 1$ and $T_{max} \gg T_0$

$$T_{max} \approx \left[\frac{\eta \tau I_0^2}{2\mu p}\sqrt{\frac{\pi}{2}}\right]^{1/\beta}\qquad (21)$$

This is the very important result, showing how the maximum temperature depends on the parameters of the super-conducting wire, current, and quench time. It is not obvious that the dimension of this result is equal to Kelvin, therefore we show this explicitly:

$$\frac{\left[\frac{\Omega}{m}\right]\left[s\right]\left[A^2\right]}{\left[\frac{kg}{m}\right]\left[\frac{J}{kg\cdot K^\beta}\right]} = \frac{\left[\frac{J}{A^2}\right]\left[A^2\right]}{J}K^\beta = K^\beta .$$

Compare (21) to the results, computed in [**6**], using the 3D model (**Figure 6**). High temperature specific resistance of a superconducting material itself is very high. For instance, for NbTi $r \sim 5\cdot10^{-7}$ Ohm·m at $T \sim 100$ K, whereas for the copper cladding $r \sim 10^{-10}$ Ohm·m [**4**]. Therefore, high-resistance section of the superconducting filament will be bypassed by surrounding copper, so that the values for η and μ in (21) should be those for the copper. We used the following set of parameters, appropriate to the result described in [**6**]:
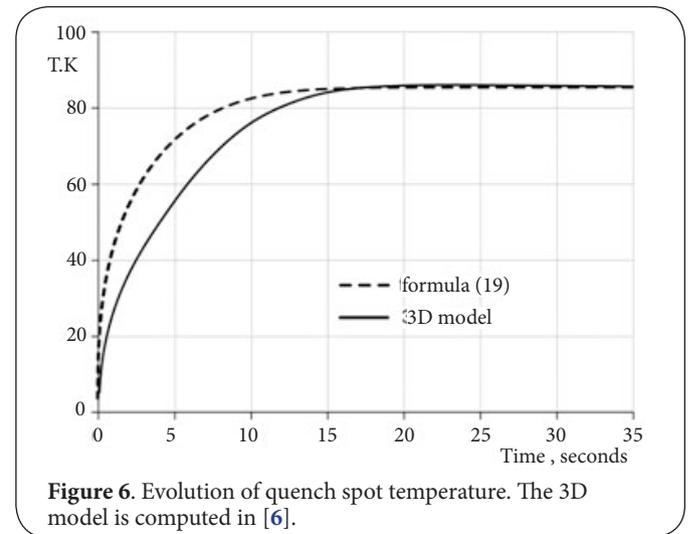

Figure 6. Evolution of quench spot temperature. The 3D model is computed in [**6**].

$I_0 = 232$ A;
$r = 1.0\cdot10^{-10}$ Ohm·m - specific resistance of copper at $T \sim 100$ K;
$S = 1.862$ mm^2 - cross section of a copper strip;
$\eta = r / S = 5.4\cdot10^{-5}$ Ohm/m;
$\tau = 12$ s - as determined from **Figure 3**;
$\rho = 8.96\cdot10^3$ kg/m^3 - density of copper;
$\mu = \rho S = 0.0167$ kg/m for copper strip;
$\alpha = 0.03$ J/(kg×K^3) as in **Figure 5**;
$\beta = 3$;
$T_0 = 4.2$ K.

Direct substitution of the above parameters into (21) gives

T_{max} = 51 K, which is not very far from the experimental value $T_{max} \approx$ 85 K [6]. The largest part of uncertainty lies in the ratio η / μ. The above values of η and μ give η / μ =0.003 in SI units. Correcting it to 0.015, we obtain necessary T_{max} =85 K. With this only correction, theoretical curve of formula (19) is shown in **Figure 6**.

Although entire qualitative behavior of the analytical formula preserves the essential features of the spot temperature dynamics, like steep initial trend and smooth finish to a stationary value of T_{max}, it differs from numerical computations in the middle part of the interval. Nonetheless, analytical formulas (19)-(21) combine all the parameters together, explicitly showing the influence of each of them - a feature that cannot be obtained by numerical computations.

Conclusion

Analytical formula derived for temporal evolution of the quench current is in good quantitative agreement with experimental data. This formula establishes relation between the quench time and very few basic parameters of the superconducting magnet: wire resistance per unit length, magnet inductance, and longitudinal quench speed. The strength of this solution is not the simplicity of the Gaussian function that describes evolution of the current, but that it establishes quantitatively correct general analytical relationship between the current and only three basic measurable parameters of the superconducting magnet.

Using the formula for the quench current, longitudinal quench velocity can be estimated from the recorded log-file of the quench event. For that, the Gaussian function should be fitted into the experimental curve with only one fitting parameter: the quench time. Quench velocity turned out to be inversely proportional to the quench time squared.

Analytical formula for temperature dynamics is in qualitative agreement with experimental data. From it, the maximum value of the temperature increase during quench can be estimated. The estimate of the maximum temperature determines probability of whether or not the superconducting coil has been damaged during quench. This very initial assessment, done quickly and without applying cumbersome numerical computations, may be important for making initial decision after quench has happened.

The overall conclusion is that even very simple analytical analysis, accounting for only few driving physical considerations, may provide valuable initial insight into rather complicated problem.

Competing interests

The author declares that he has no competing interests.

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